Gauge semi-simple extension of the Poincaré group

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Abstract

Based on the gauge semi-simple tensor extension of the D-dimensional Poincaré group another alternative approach to the cosmological term problem is proposed.

PACS: 02.20.Sv; 11.30.Cp; 11.15.-q

Keywords: Poincaré algebra, Tensor, Extension, Casimir operators, Gauge group

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1. Recently the approach to the cosmological constant problem based on the tensor extension of the Poincaré algebra with the generators of the rotations M_{ab} and translations P_a [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]

$$[M_{ab}, M_{cd}] = (g_{ad}M_{bc} + g_{bc}M_{ad}) - (c \leftrightarrow d), \tag{1}$$

$$[M_{ab}, P_c] = g_{bc}P_a - g_{ac}P_b, \tag{2}$$

$$[P_a, P_b] = cZ_{ab}, \tag{3}$$

$$[M_{ab}, Z_{cd}] = (g_{ad}Z_{bc} + g_{bc}Z_{ad}) - (c \leftrightarrow d), \tag{4}$$

$$[P_a, Z_{bc}] = 0,$$

$$[Z_{ab}, Z_{cd}] = 0$$

was given by de Azcarraga, Kamimura and Lukierski [20]. Here Z_{ab} is a tensor generator, g_{ab} is a constant Minkovski metric and c is some constant.

In this paper we present another approach to the problem based on the gauge semi-simple tensor extension of the D-dimensional Poincaré group which Lie algebra has the following form [13, 21]:

$$[Z_{ab}, P_c] = \frac{\Lambda}{3c}(g_{bc}P_a - g_{ac}P_b), \tag{5}$$

$$[Z_{ab}, Z_{cd}] = \frac{\Lambda}{3c}[(g_{ad}Z_{bc} + g_{bc}Z_{ad}) - (c \leftrightarrow d)], \tag{6}$$

whereas the form of the rest permutation relation (1)-(4) is not changed. Λ is some constant.

The Lie algebra (1)-(6) has the following quadratic Casimir operator:

$$P^aP_a + cZ^{ab}M_{ba} + \frac{\Lambda}{6}M^{ab}M_{ab} \stackrel{\text{def}}{=} X_kh^{kl}X_l,$$

where $X_k = \{P_a, M_{ab}, Z_{ab}\}$ is a set of the generators for the Lie algebra under consideration (1)-(6) and the tensor h^{kl} is invariant with respect to the adjoint representation

$$h^{kl} = U^k{}_mU^l{}_nh^{mn}.$$

The inverse tensor h_{kl} ($h_{kl}h^{lm} = \delta_k{}^m$) is invariant with respect to the co-adjoint representation

$$h_{kl} = h_{mn}U^m{}_kU^n{}_l.$$

2. Let us consider a gauge group corresponding to the Lie algebra (1)-(6). To this end we introduce a gauge 1-form

$$A = A^kX_k = dx^\mu(e_\mu{}^aP_a + \frac{1}{2}\omega_\mu{}^{ab}M_{ab} + \frac{1}{2}B_\mu{}^{ab}Z_{ab})$$

with the following gauge transformation:

$$A' = G^{-1}dG + G^{-1}AG,$$

where G is a group element corresponding to the Lie algebra (1)-(6). Here x^μ are space-time coordinates, $e_\mu{}^a$ is a vierbein, $\omega_\mu{}^{ab}$ is a spin connection and $B_\mu{}^{ab}$ is a gauge field conforming to the tensor generator Z_{ab}.

A contravariant vector F^k of the field strength 2-form

$$F = F^k X_k = dA + A \wedge A = \frac{1}{2} dx^\mu \wedge dx^\nu F_{\mu\nu}$$

is transformed homogeneously under the gauge transformation

$$F'^k X_k = U^k{}_l F^l X_k = G^{-1} F^k X_k G.$$

The field strength

$$F_{\mu\nu} = F_{\mu\nu}{}^k X_k = \partial_{[\mu} A_{\nu]} + [A_\mu, A_\nu]$$

has the following decomposition:

$$F_{\mu\nu} = F_{\mu\nu}{}^a P_a + \frac{1}{2} R_{\mu\nu}{}^{ab} M_{ab} + \frac{1}{2} F_{\mu\nu}{}^{ab} Z_{ab}.$$

Here

$$F_{\mu\nu}{}^a = T_{\mu\nu}{}^a + \frac{\Lambda}{3c} B_{[\mu}{}^{ab} e_{\nu]b},$$

where

$$T_{\mu\nu}{}^a = \partial_{[\mu} e_{\nu]}{}^a + \omega_{[\mu}{}^{ab} e_{\nu]b}$$

is a torsion,

$$R_{\mu\nu}{}^{ab} = \partial_{[\mu} \omega_{\nu]}{}^{ab} + \omega_{[\mu}{}^{ac} \omega_{\nu]c}{}^b$$

is a curvature tensor and

$$F_{\mu\nu}{}^{ab} = \partial_{[\mu} B_{\nu]}{}^{ab} + \omega_{[\mu}{}^{c[a} B_{\nu]}{}^{b]}{}_c + \frac{\Lambda}{3c} B_{[\mu}{}^{ca} B_{\nu]}{}^b{}_c + c e_{[\mu}{}^a e_{\nu]}{}^b$$

is a component corresponding to the tensor generator Z_{ab}.

An invariant Lagrangian has the following form:

$$L = -\frac{e}{4} h_{kl} F_{\mu\nu}{}^l F_{\rho\lambda}{}^k g^{\mu\rho} g^{\nu\lambda} = \frac{e}{4} \left(\frac{1}{c} R_{\mu\nu}{}^{ab} F_{\rho\lambda;ab} + \frac{\Lambda}{6c^2} F_{\mu\nu}{}^{ab} F_{\rho\lambda;ab} - F_{\mu\nu}{}^a F_{\rho\lambda;a} \right) g^{\mu\rho} g^{\nu\lambda}.$$

Note that there exists a curious limit $c \to \infty$ which results in

$$L \to \mathcal{L} = \left(\frac{1}{2} R + \Lambda - \frac{1}{4} T_{\mu\nu}{}^a T^{\mu\nu}{}_a \right) e,$$

where $R = R_{\mu\nu}{}^{ab} e_a{}^\mu e_b{}^\nu$ is a scalar curvature, $g^{\mu\nu} = g^{ab} e_a{}^\mu e_b{}^\nu$ is a metric tensor, $e = \det e_\mu{}^a$ is a determinant of the vierbein and Λ is a cosmological constant.

3. Thus, we have presented another alternative approach to the cosmological term problem within the gauge semi-simple tensor extension of the Poincaré group.

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